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The Golden Ratio

The Golden Ratio is sometimes called the *Golden Section* or the *Divine Proportion*. Three points A, B and C divide a line in this proportion if $\frac{AC}{AB} = \frac{AB}{BC}$



This ratio is sometimes called ϕ and has a value of about 1.618. If we let $BC = 1$ it follows that $AB = \phi$ and $AC = \phi^2$. Since $AC = AB + BC$ we deduce that $\phi^2 = \phi + 1$.

By solving the quadratic we find $\phi = \frac{\sqrt{5} + 1}{2}$

Artists and architects have long held that rectangles with sides in this ratio offer the most pleasing aspect ratio. If only the designers of 16:9 TV screens knew this.

The Golden Ratio is constantly at hand. You may have watched a documentary on SBS which describes the layout of buildings on the Acropolis. The Golden Ratio was used extensively to derive the proportions of the Parthenon and the detail of the Erechtheion.

Many of you, particularly subscribers to the *AAMT-List*, will have recently bought a copy of *Donald in Mathmagicland* and found that it includes a section about the Golden Ratio and the ratios within a five pointed star or pentagram. You may have relaxed during your vacation reading *The Da Vinci Code* where you would have read a more intriguing interpretation of these matters.

Here are two computing exercises which provide follow up activities after your class watches *Donald in Mathmagicland*.

The Golden Ratio is closely linked with the Fibonacci sequence $\{1, 1, 2, 3, 5, \dots\}$ for which new terms are generated by adding the two previous terms. If we count the clockwise and counter-clockwise spirals formed by the seeds of sunflowers or pinecones the numbers are consecutive Fibonacci numbers and new seeds are offset at an angle of $2\pi/\phi$. The convergence of the ratios of consecutive Fibonacci numbers to the Golden Ratio suggests this simple spreadsheet exercise.

This spreadsheet shows the Fibonacci sequence in Column A [copy down $A3 = A1 + A2$]. The quotients of consecutive terms converge to $\phi = 1.618\dots$ in Column B [copy down $B1 = A2/A1$].

| | A | B |
|----|----------|------------------|
| 1 | 1 | 1.00000000000000 |
| 2 | 1 | 2.00000000000000 |
| 3 | 2 | 1.50000000000000 |
| 4 | 3 | 1.66666666666667 |
| 5 | 5 | 1.60000000000000 |
| 6 | 8 | 1.62500000000000 |
| 7 | 13 | 1.61538461538462 |
| 8 | 21 | 1.61904761904762 |
| 9 | 34 | 1.61764705882353 |
| 10 | 55 | 1.61818181818182 |
| 11 | 89 | 1.61797752808989 |
| 12 | 144 | 1.61805555555556 |
| 13 | 233 | 1.61802575107296 |
| 14 | 377 | 1.61803713527851 |
| 15 | 610 | 1.61803278688525 |
| 16 | 987 | 1.61803444782168 |
| 17 | 1597 | 1.61803381340013 |
| 18 | 2584 | 1.61803405572755 |
| 19 | 4181 | 1.61803396316671 |
| 20 | 6765 | 1.6180339852180 |
| 21 | 10946 | 1.61803398501736 |
| 22 | 17711 | 1.61803399017560 |
| 23 | 28657 | 1.61803398820532 |
| 24 | 46368 | 1.61803398895790 |
| 25 | 75025 | 1.61803398867044 |
| 26 | 121393 | 1.61803398878024 |
| 27 | 196418 | 1.61803398873830 |
| 28 | 317811 | 1.61803398875432 |
| 29 | 514229 | 1.61803398874820 |
| 30 | 832040 | 1.61803398875054 |
| 31 | 1346269 | 1.61803398874965 |
| 32 | 2178309 | 1.61803398874999 |
| 33 | 3524578 | 1.61803398874986 |
| 34 | 5702887 | 1.61803398874991 |
| 35 | 9227465 | 1.61803398874989 |
| 36 | 14930352 | 1.61803398874990 |
| 37 | 24157817 | 1.61803398874989 |

This simple *JavaSketchpad* exercise explores the Golden Ratios within a five pointed star. The advantage of *JavaSketchpad* is that students can use it on their home computers free of charge. They need to download the jsp4.JAR applet from www.keypress.com/sketchpad and success will depend on the version of Java used in their machine's operating system. It all works fine on recent machines with new browser software.

Students should open to the source code of a blank browser screen and type in the code shown below. The file should then be saved as an html file in the same folder as the jsp4.JAR applet. On opening the html file the five pointed star should appear. The active point is J and wherever J is moved relative to the centre C, the ratios of the segments along the line JL remain constant with a value of $\phi = 1.618...$

```
<HTML>
<HEAD>
<TITLE>Pentagram</TITLE>
</HEAD>
<BODY>
<APPLET codeBase=jsp height=310 width=250
archive=jsp4.JAR align=left code=GSP.class>
<PARAM NAME="Frame" VALUE="1">
<PARAM NAME="MeasureInDegrees" VALUE="1">
<PARAM NAME=BackRed Value=255>
<PARAM NAME=BackGreen Value=255>
<PARAM NAME=BackBlue Value=255>
<PARAM NAME="DirectedAngles" VALUE="0">
<PARAM NAME="Construction" VALUE="
{1} Point (120,120) [label('C')];
{2} Point (120,220) [label('J')];
{3} Circle (1,2);
{4} Rotation (2,1,1.256637) [label('K')];
{5} Rotation (4,1,1.256637) [label('L')];
{6} Rotation (5,1,1.256637) [label('M')];
{7} Rotation (6,1,1.256637) [label('N')];
{8} Segment (2,5);
{9} Segment (2,6);
{10} Segment (4,6);
{11} Segment (4,7);
{12} Segment (5,7);
{13} Intersect (10,12) [label('P')];
{14} Intersect (9,12) [label('Q')];
{15} Intersect (9,11) [label('R')];
{16} Intersect (8,11) [label('S')];
{17} Intersect (8,10) [label('T')];
{18} Segment (2,17);
{19} Segment (17,5);
{20} Segment (16,17);
{21} Ratio/Segments ( 8,18,50,250,'Ratio JL/JT = ');
{22} Ratio/Segments (18,19,50,270,'Ratio JT/JS = ');
{23} Ratio/Segments (19,20,50,290,'Ratio JS/ST = ');
">
</APPLET>
</BODY></HTML>
```

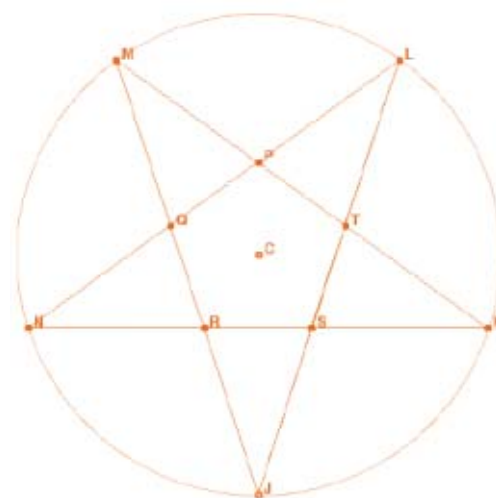
The <APPLET> line allows us to adjust the dimensions of the frame to fit a particular screen. Construction items are numbered {1}, {2} etc. The first item, {1}, is the point C at the centre of the circle. Point J, item {2}, is then defined below C.

Item {3} is coded circle (1,2) which means its centre is at C, {1}, and it passes through J, {2}.

Item {4} defines a new point K which is obtained by rotating J anticlockwise about C through the angle 72° . But, even though MeasureInDegrees is switched on with VALUE="1", we have to express the angle 72° in radian measure ($2\pi/5 \approx 1.256637$).

Similarly we define L, M and N with further rotations by 72° . From {8} to {12} we draw the segments joining J, K, L, M and N and from {13} to {17} their intersection points at P, Q, R, S and T. Then we join the points JL, JT and JS so that each of these segments has an item number. Finally we evaluate the ratios of these segments and find that they are each equal to the Golden Ratio. You could use *Cabri* or *GSP* just as easily. Although the Java Applet can be used within the same html code as JavaScript, I do not know of any way to pass JavaScript values to the Java applet.

JavaSketchpad provides a simple way to describe such constructions in this format and it has the added advantages of being free and easily shared using the Internet. Using *JSP* our students are much more likely to explore these ideas at home and catch something of Da Vinci's and Disney's fascination with these topics.



Ratio JL/JT = 1.618
Ratio JT/JS = 1.618
Ratio JS/ST = 1.618

